CS 2200 Assignment - 2

Instructor Avah Banerjee
Due Date. Oct 18 12:00 Noon

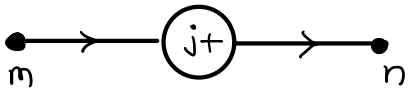
Your answers should not contain any handwritten parts. All relevant written sections should be typed and compiled into a single PDF, including screenshots, code, and figures where applicable.

Problem 1 (25 Pts) Create a Turing machine (draw the full state transition diagram as we did for the TM that accepts strings with 2^n 1's in class) that accepts all strings over $\Sigma = \{0, 1\}$ (i.e., binary strings) such that the total number of 0's (n_0) and the total number of 1's (n_1) sum to 3^k (i.e., $n_0 + n_1 = 3^k$) for some non-negative integer k, and rejects every other string. For example, the string 011 is accepted, but 11 is rejected. Assume the tape alphabet is $\{0, 1, \sqcup\}$.

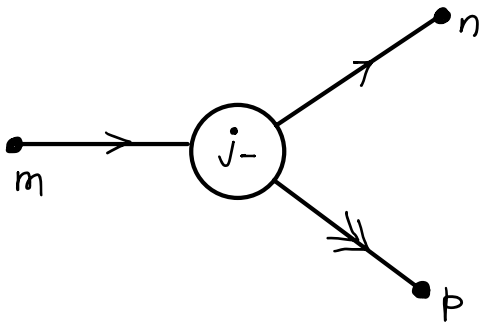
Problem 2 (30 Pts) Create a Turing machine with two non-halting states, one halting state, and n symbols (say $s_1, \ldots, s_n$), as well as the special blank symbol $\sqcup$, which halts after at least $4n - 1$ steps. Show the state transition diagram and explain why it runs for at least that many steps.

Problem 3 (30 Pts) A *Minsky machine* [1] consists of a finite set of registers, $r_1, r_2, \ldots, r_k$, each capable of holding an arbitrary non-negative integer, and a *program* made up of *orders* of one of two types.

The first type has the form:



The interpretation is that at point m in the program, register r_j is incremented by one, and execution proceeds to point n in the program.

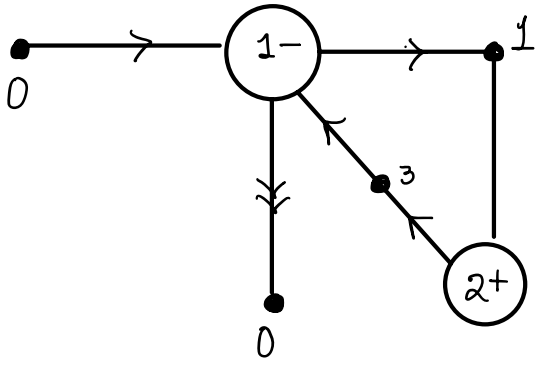


The second type of order has the form:

The interpretation is that at point m in the program, register r_j is decremented if it contains a positive integer, and execution proceeds to point n in the program. If register r_j is zero, then execution simply proceeds to point p in the program.

The *program* for the Minsky machine consists of a collection of such orders, in a form shown in the figure below. The starting point and all possible halting points for the program are conventionally labeled zero. The above program takes the contents of register r_1 and adds them to register r_2, while decrementing r_1 to zero.

Create a Minsky machine that computes the square of a natural number.



Problem 4 (15 Pts) Describe two different Turing machines, M and N, where M outputs ⟨N⟩ and N outputs ⟨M⟩, when started on any input.

References

[1] Nielsen, M. A., & Chuang, I. L. (2010). Quantum computation and quantum information. Cambridge university press.