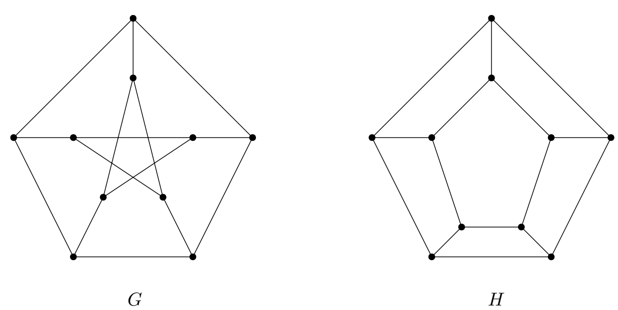
CS 2200 Assignment - 3

Instructor Avah Banerjee
Due Date. Nov 18 12:00 Noon

Your answers should not contain any handwritten parts. All relevant written sections should be typed and compiled into a single PDF, including screenshots, code, and figures where applicable.

Problem 1 (20 Pts) The EDGECOVER problem is defined as follows: Given a simple undirected graph $G = (V, E)$ ($|V| = n, |E| = m$) and a positive integer k, the task is to determine whether there exists a subset $E' \subset E$ of edges of size at most k such that every vertex in G is an endpoint of some edge in E'. Show that EDGECOVER $\in$ P. [Hint: If you can compute the maximum matching for the graph G, you can use it to solve the EDGECOVER problem. A matching in a graph is a collection of edges that are pairwise disjoint. Assume that there is a polynomial-time algorithm to find a maximum matching.]

Problem 2 (20 Pts) Provide a proof that the following two graphs are non-isomorphic. One straightforward proof would be to show that no isomorphism exists between the graphs by exhaustively checking all possible isomorphisms and demonstrating that each fails for some edge. However, such a proof would be too lengthy, as there are 10! possible isomorphisms between them. Your task is to construct a more concise proof.

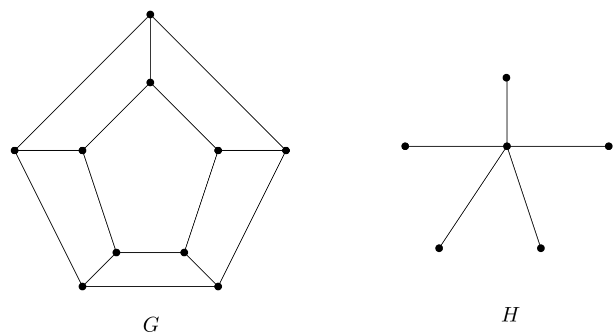


Problem 3 (15 Pts) The PARTITION problem is defined as follows: Given a set of n positive integers $S = \{s_1, s_2, \ldots, s_n\}$, determine whether there exists a subset $S' \subseteq S$ such that the sum of the elements in S' is equal to the sum of the elements in $S \setminus S'$. In other words, can the set S be partitioned into two subsets with equal sums? Show that PARTITION $\leq_p$ SUBSETSUM.

Problem 4 (25 Pts) **Graph Minor**: A graph H is a minor of a graph G if H can be formed from G by a series of operations involving:

- Deletion of vertices and edges.
- Contraction of edges (merging the two vertices connected by an edge and combining their incident edges).

For example, in the figure below, H is a minor of G. The GRAPHMINOR problem is to



determine, given two simple undirected graphs $G = (V, E)$ and H as input, whether H is a minor of G. Show that GRAPHMINOR $\in$ NP. Here we assume G has n vertices and m edges.

Problem 5 (20 Pts) Consider the following problem:
FIB $= \{(n, k) \mid n$ and k are positive integers and the n^{th} Fibonacci number $\leq k\}$.
Is FIB $\in P$? Justify your answer. [The input to the problem is a binary encoding of n and k.]