CS 2200 Assignment - 4

Instructor Avah Banerjee
Due Date. Dec 09 12:00 Noon

Your answers should not contain any handwritten parts. All relevant written sections should be typed and compiled into a single PDF, including screenshots, code, and figures where applicable.

Problem 1 (30 Pts) Determine, for each of the following deterministic finite automata, the language it recognizes. All these languages are over the alphabet $\{a, b\}$.

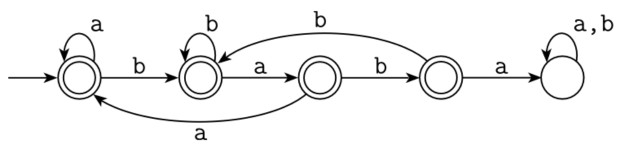

Figure 1: DFA-1

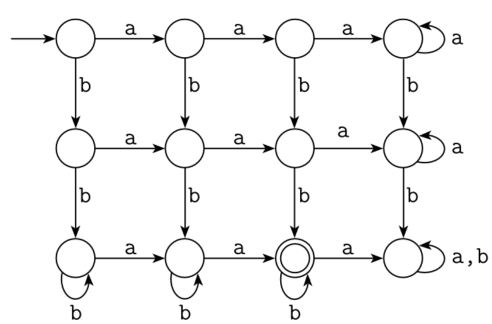

Figure 2: DFA-2

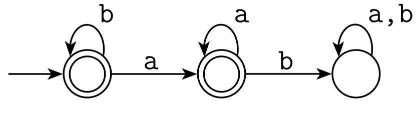

Figure 3: DFA-3

Problem 2 (20 Pts) Let $\Sigma = \{0, 1, +, =\}$ and

$$ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

Show that ADD is not regular.

Problem 3 (20 Pts) Consider a probabilistic finite automaton over the alphabet $\{a, b\}$. It has two transition functions, δ_1 and δ_2, and three states: q_0, q_1, and q_2 (the accepting state). The transition function δ_1 is defined as follows:

$$\delta_1(q_0, a) = q_0, \quad \delta_1(q_0, b) = q_1, \quad \delta_1(q_1, a) = q_1, \quad \delta_1(q_1, b) = q_2, \quad \delta_1(q_2, a/b) = q_2.$$

The transition function δ_2 is the same as δ_1 for the states q_1 and q_2, but differs for the state q_0:

$$\delta_2(q_0, a) = q_1 \quad \text{and} \quad \delta_2(q_0, b) = q_0.$$

At each step, the automaton chooses one of the two transition functions by flipping a fair coin.

1. If two strings $a^n b^m a^p$ and $a^r b^s a^t$ are accepted with the same probability, what are the possible relationships between m, n, p and r, s, t. All integers are assumed non-negative?

2. What is the probability of accepting a string of the form $a^n b a^p$ and $n \geq 0, p > 0$?

Problem 4 (30 Pts) Let $\mathbb{F}_7$ be a set (more technically, a field) containing $\{0, 1, 2, 3, 4, 5, 6\}$, where addition and multiplication (in decimal) are performed modulo 7. For example, $2 \cdot 5 = 10 \equiv 3 \pmod 7$, and the same applies to the addition operation.

Consider the following polynomial:

$$p(x, y, z) = x^2 z - xyz + y^2 x + z^3.$$

Find a set S of size 4 (you can either do this by hand or write a short program) such that the probability of a random (uniformly and independently chosen) assignment of values to the three variables from S making $p(x, y, z) \equiv 0 \pmod 7$ is minimized. What is the probability?