CS 5001 Homework - 2

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Problem 1 (Single-qubit rotations). Consider the single-qubit gates given by the following unitaries:

$$V_1 = \frac{1}{2} \begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}, \quad V_2 = \begin{pmatrix} e^{i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix}$$

(a) Determine the axis of rotation on the Bloch sphere for each unitary.

(b) Calculate the rotation angle (in radians) for each gate.

(c) Briefly explain the reasoning behind your deductions based on the provided matrices.

Problem 2 Determine the axis and angle of rotation for the two single-qubit unitaries, A and B, such that

$$ABA^{\dagger}B^{\dagger} = \sigma_X.$$

Problem 3 (A variation of CHSH game). If the CHSH game were modified so that Alice and Bob aim to satisfy $a \vee b = x \oplus y$ instead, what classical and quantum strategies could they employ, and what would be their maximum winning probabilities?

Problem 4 (Geometric interpretation of controlled rotations). Consider the controlled-X rotation gate $C(R_X(\gamma))$ defined as:

$$C(R_X(\gamma)) = |0\rangle \langle 0| \otimes I + |1\rangle \langle 1| \otimes R_X(\gamma),$$

where $\gamma \neq \pi$. Provide a clear geometric interpretation of the effect of this gate on a two-qubit system. Specifically, describe how the second qubit is rotated on the Bloch sphere depending on the state of the first (control) qubit.